A Novel Continuous Visual Analog Scale Model Derived from Pain-relief Demand Index via Hilbert Huang Transform for Postoperative Pain

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Abstract

In this investigation, we proposed a continuous pain intensity scaling model derived from an innovative parameter of pain-relief demand (PRD) index for acute postoperative patients of gynecology using patient-controlled analgesia (PCA) via the occurrence frequency of pain-relief demands. To derive PRD, a simplified fluctuation time series of pain feeling was simulated according to the distribution of pain-relief demands using an exponential decay function. Furthermore, we applied the Hilbert Huang transform to obtain the time-amplitude distribution of the first intrinsic mode function (IMF) of fluctuation time series and defined the PRD index using a function of amplitude. A total of 2466 visual analogue scale (VAS) values derived from interviews with 470 postoperative patients at various time points were compared with this PRD index extracted from the PCA records of the same patients. According to the statistical result of one-way analysis of variance (ANOVA, P < 0.001), VAS was significantly linearly dependent on the PRD index. Although the result of this VAS model is not a perfect linear relationship between VAS and PRD index (r = 0.490), it is significantly better than the result of our previous study using a novel fuzzy pain demand index (FPD). This result shows this approach is promising for further study.

Keywords: Visual analog scales (VAS), Patient-controlled analgesia (PCA), Hilbert huang transform (HHT), Intrinsic mode function (IMF), Fuzzy pain demand index (FPD)

1. Introduction

Pain is a complex, private experience, and attempts to make valid assessments of it have been fraught with difficulties [1,2]. The wide variation in the individual pain experiences of patients leads to a large variability in the pain scale ratings. In addition, pain scale measurements are often interpreted in various ways by different researchers and clinicians, depending on the criteria they choose to apply [2]. Therefore, pain is a multidimensional scale which is affected by both sensory-discriminative and emotional-cognitive components of a patient’s suffering [3,4]. In clinical settings, the most commonly used measures of pain intensity, including visual analog scales (VASs), numerical rating scales (NRSs), and verbal rating scales (VRSs) have been shown to have adequate sensitivity to changes in pain associated with treatment across many populations and settings [5-9]. However, all these scales require intervention by other people (i.e., medical doctors or nurses) to ask the patient about results. Hence, interviews cannot be conducted frequently because they are time-consuming for medical staffs and bothersome for patients. Moreover, pain had been proposed as “the fifth vital sign” following the four basic vital signs of core temperature, blood pressure, pulse and respiration rates [10,11]. Unfortunately, there is still no objective and promising measurement for evaluating pain intensity. So, it is still very difficult to on-line analyze pain intensity as is done for the four basic vital signs.

In clinical practices of pain management, VAS is a reliable standard for assessing pain. VAS provides a scoring meter, which is a 100-mm line anchored by words “no pain” at the end of the line marked by 0 mm and “the most pain I can imagine” at the other end of the line marked by 100 mm, for patients to rate their subjective pain feeling. For the purpose of defining a reliable, objective and continuous pain scale for on-line monitoring postoperative pain, Shieh et al. indicated that records stored in patient-controlled analgesia (PCA) devices contain important information about self-reported pain intensity [12,13]. In this investigation, we present a new approach to

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evaluate the continuous pain intensity of postoperative patients without disturbance and to reduce the workload on medical staff. This new VAS model was derived from an innovative parameter of pain-relief demand (PRD) index for the postoperative patients of gynecology using PCA via the occurrence frequency of pain-relief demands.

The PRD index was based on the occurrence frequency of pain-relief demands via the recordings of PCA in this investigation. The assumption was that a patient always presses the button of pain-relief demand at the instant when the pain feeling is over the temporary limitation of individual pain bearing. Here, we assume that both pain feeling and limitation of individual pain bearing are subjective and temporary. During the period that the patient suffers severe pain, his temporary limitation of pain bearing is respectively higher than that under the situation that the patient suffers mild pain. Therefore, the pain scale should be various for each instant of button-pressing event for pain-relief demand. However, the occurrence frequency of pain demand may be a totally different parameter to quantify the pain intensity. Generally, a higher occurrence frequency for pain-relief demand is driven by higher pain intensity. Therefore, the pain scale should be a function of occurrence frequency of pain-relief. However, the occurrences of pain-relief demand are discrete but not continuous events, so the time series of occurrence frequency can be derived only via particular algorithms of data processing. To derive a continuous time series of occurrence frequency of pain-relief, we simulate a fluctuation time series of pain feeling of the patient via the occurrence of pain-relief demand by an exponential decay function. Furthermore, since the simulated time series of pain feeling is nonlinear and non-stationary, an innovative signal analysis algorithm of Hilbert Huang transform (HHT), proposed by Huang et al. [14], was applied to derive the time-amplitude distribution according to the fluctuation pattern of the simulated time series of pain feeling. Furthermore, the amplitude was defined by the exponential decay function of occurrence interval of pain-demand, which is the inverse of occurrence frequency of pain-relief demand in the simulated time series of pain feeling.

Therefore, the present study had two primary goals. The first was to propose an innovative parameter of PRD index derived from pain-relief demand to model this multidimensional postoperative pain. We wished to know how this highly nonlinear pain demand pattern signals is applied in generating PRD index via HHT algorithm. The second goal of this investigation was to generate a continuous VAS model derived from PRD index. We wished to know how this model of the VAS was generated via linear regression algorithm and how this model was related to PRD index at each level of VAS.

2. Materials and methods

2.1 Simulating the fluctuating time series of pain feeling according to the occurrence of pain-relief demand

As a typical postoperative pain management, PCA provides patients the device of self-controlled analgesia. Patients can respond to the requirements of pain relief by pressing the pushbutton of demand when it is necessary. If the time interval between time points of current and preceding demands is over the lockout period, a bolus (i.e., a constant dose of pain killer, e.g., 1 ml of morphine) is given for analgesia. In medical practice, the constant dose of pain-killer and the lockout period are pre-set by the medical staff based on their experiences of pain management. However, whether the scenario of pain management is appropriate or not is the critical issue medical staff needed to know. Therefore, an objective continuous pain scale would be helpful to answer this issue.

As mentioned above, pain is a subjective and complex feeling depended on individual experience. It is not precise to define a constant pain scale at the time point of the occurrence of each pain-relief demand. We believe the occurrence frequency of pain-relief demand is another critical clue with which to evaluate the pain intensity. The problem is how to define a continuous pain scale based on the occurrence frequency of pain-relief demand. Thus, we made our efforts to simulate the fluctuating time series of pain feeling via the occurrence of pain-relief demand and to evaluate pain intensity via the fluctuating pattern of simulated time series of pain feeling.

To accomplish the simulation of pain feeling via the occurrence of pain-relief demands, we assumed the value of pain feeling for the threshold of pain bearing was 1. When the patient’s pain feeling approaches and crosses the threshold of pain bearing, he presses the pushbutton to respond to the demand for pain relief. Then, the pain feeling of patient is psychologically comforted because of the expression of pain-relief demand. This postpones the appearance of the next pain-relief demand. The patient’s pain feeling is relatively lower than the threshold of pain bearing until the demand of pain relief occurs again. Between the time points of occurrences for two consecutive pain-relief demands, pain feeling is simulated using the symmetrical exponential decay function, shown as follows:

\[
\begin{align*}
  f &= 1 & t &= t_0, t_1 \\
  f &= e^{-(t-t_0)/\tau_1} & t &< t_0 + \tau_1/2 \\
  f &= e^{-(t-t_1)/\tau_1} & t &\geq t_0 + \tau_1/2
\end{align*}
\]

where \( f \) is the pain feeling, \( t_0, t_1 \) are the time points of occurrences for two consecutive pain-relief demands; and \( \tau \) is the decay coefficient.

The fluctuating time series of pain feeling according to the occurrences of pain-relief demands was simulated and is shown in Fig. 1. Here, Fig. 1(a) shows an example of occurrence time series of pain-relief demands, and Fig. 1(b) shows the simulated fluctuating time series of pain feeling via the occurrence time series of pain-relief demands shown in Fig. 1(a).

2.2 Deriving the time-amplitude of the fluctuating pattern of pain feeling using Hilbert transform

Empirical mode decomposition (EMD) is a promising algorithm to decompose the intrinsic mode functions (IMF) from a non-stationary time series adaptive to the nature of data. Moreover, the original time series can be reconstructed by summing the decomposed IMFs and the final residue after decomposition processes as the following equation:
\[ f(t) = \sum_{n=1}^{N} X_n(t) + r_n \]  

(2)

where \( f(t) \) is the original time series, \( X_n \) is the \( n \)th IMF, and \( r_n \) is the residue after \( n \) decomposition operations.

Since the simulated time series of pain feeling is a highly non-stationary time series, we therefore applied the algorithm of EMD to extract the first IMF from the fluctuating time series of pain feeling. However, since the IMF is a periodic mode function, it is necessary to conduct a further signal processing to derive the time-amplitude distribution of the fluctuating pattern of pain feeling. Hence, for a non-stationary periodic mode function, Hilbert transfer is a promising algorithm to derive the analytical mode function [15,16]. In Hilbert transfer, the conjugate part (i.e., \( Y(t) \)) of the analytical mode function can be derived from the original time series of the intrinsic mode function, \( X(t) \), by Hilbert transform, as

\[ Y(t) = \frac{1}{\pi} P \int_{\infty}^{-\infty} \frac{X(\tau)}{t-\tau} d\tau \]  

(3)

where \( P \) indicates the Cauchy principal value. This transform exists for all functions of class \( Lp \) [17,18]. Therefore, the analytic time series, \( Z(t) \), is formed using the complex conjugate time series of \( X(t) \) and \( Y(t) \), as

\[ Z(t) = X(t) + jY(t) = a(t)e^{j\theta(t)} \]  

(4)

in which, the amplitude, \( a(t) \), and phase, \( \theta(t) \), are shown as the following equations.

\[ a(t) = \sqrt{X^2(t) + Y^2(t)} \quad \text{and} \quad \theta(t) = \tan^{-1} \left( \frac{Y(t)}{X(t)} \right) \]  

(5)

Thus, Hilbert transfer promises us to derive the time-amplitude distribution of a non-stationary periodic time series. Figure 2 shows an illustration of the process from the fluctuating time series of pain feeling to the time-amplitude distribution of fluctuating pattern. Figure 2(a) shows the fluctuating time series of pain feeling. Figure 2(b) shows the first IMF decomposed from the fluctuating time series of pain feeling. Figure 2(c) shows the time-amplitude distribution of the first IMF.

2.3 Definition of the new pain scale of PRD index

Before we make the definition of the new pain scale of PRD index, we should recall the assumption and the simulation we take for pain feeling mentioned above. In our assumption, PRD index is assumed to be a function of the occurrence frequency of pain-relief demands. However, since the occurrence of pain-relief demand is discrete, we take a simulation of pain feeling and extract the fluctuating pattern of pain feeling by the algorithm of EMD. Furthermore, the fluctuating pattern of pain feeling was interpreted by Hilbert transfer to derive a continuous time-amplitude distribution of fluctuating pattern of pain feeling. According to the definition of pain feeling simulation shown in equation (1), the fluctuation of pain feeling is proportional to the logarithmic time interval between two occurrences of pain-relief demands. Since IMF 1 was decomposed from function of pain feeling, amplitude of IMF 1 should be proportional to the fluctuation of pain feeling and logarithmic time interval between two occurrences of pain-relief demands. Moreover, PRD is defined...
as a function of occurrence frequency of pain-relief demand, which performs as the inverse of the time interval, so the new pain scale of PRD index should be proportional to the logarithmic inverse of fluctuating amplitude. Thus, the PRD index can be defined by the following equation (6).

\[ p(t) = \ln \frac{1}{a_i(t)} + 4 \]  (6)

where \( p(t) \) is pain-relief demand index; \( a_i(t) \) is the amplitude of the first IMF decomposed from the simulated time series of pain feeling.

PRD index adds 4 to obtain positive values for overall time series. Moreover, we also set a lower boundary of 0. Figure 3 shows an illustration of the original occurrence of pain-relief demand, simulated time series of pain feeling, and the result of the PRD index.

2.4 Comparison between VAS and PRD index using ANOVA

One-way analysis of variance (ANOVA) is a statistical algorithm suitable for comparing the response and a single factor. In this investigation, VAS was the observed response and PRD index was the single factor. \( P \)-value < 0.05 was considered statistically significant [19]. Furthermore, a linear statistical model was applied to describe the relationship between VAS and PRD index.

2.5 Pearson’s correlation coefficient

Pearson’s product-moment correlation coefficient is a measurement to identify the linear relationship between two variables [19]. It is named after Karl Pearson. The correlation coefficient \( r \) is calculated by equation (7).

\[
 r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \]  (7)

where \( r \) is Pearson’s correlation coefficient, \( x_i \) and \( y_i \) are the \( i \)th item of two variables, and \( \bar{x} \) and \( \bar{y} \) are the means of two variables.

Pearson’s correlation coefficient shows a perfect linear relationship between two variables by the value of 1 but negative correlation by the value of -1.

2.6 A linear statistical model used to verify the connection between VAS and PRD

In this study, a linear statistical model was used to fit the relationship between VAS and PRD index. The linear statistical model is presented using the means model, as in the following equation (8) [20].

\[
p_{ij} = \bar{y} + v_i + \varepsilon_{ij} \quad \{i = 1,2,\ldots,a \} \quad \{j = 1,2,\ldots,n \} \]  (8)

where, \( \bar{y} \) is the overall mean of values of PRD index and \( v_i \) is called the \( i \)th treatment effect. \( \varepsilon_{ij} \) is the random error component that incorporates all other sources of variability.

Theoretically, random errors are assumed to be normally and independently distributed with mean zero and variance \( \sigma^2 \).

When the variance is a constant for all levels of VAS, the linear statistical model is fit to describe the correlation between PRD index and VAS. Then, the relationship between VAS and PRD can be expressed by a linear equation with the following equation (9).

\[
 VAS = \alpha \times PRD + \beta \]  (9)

2.6 Experimental design

In this investigation, 470 PCA records of gynaecological patients in the i-pain database were selected as the study cases. These study cases included postoperative analgesia managements of gynecological surgeries. These patients wore PCA devices over 48 hours for self-controlled analgesia. The average age of patients was 26.8 years old, and the averaged weight was 64.6 kg. 2466 records of VAS were included in this investigation. A VAS record was obtained via a clinical interview conducted by a trained medical staff and included items of value to VAS, time of interview conducted by medical staff, and the other physiological parameters. The scale of VAS was a discrete level from 0 (e.g., no pain) to 10 (e.g., very severe pain). The number of cases for each VAS level is shown in Table 1.

3. Results

The PRD index model was applied to analyze 470 study cases of gynecological patients wearing PCA in this investigation. We also found 2466 VAS records with their corresponding time of interviews from these study cases. We derived the continuous time series of PRD index via the
occurrence time series of pain-relief demands by the pain evaluation algorithm proposed in this investigation. To derive an appropriate value of decay coefficient for equation (1), we tried different values of decay coefficients from 0.00001 to 0.5 to evaluate the performances using the Pearson’s correlation coefficients between VAS and PRD. Figure 4 shows the results of Pearson’s correlation coefficients between VAS and PRD versus the decay coefficients used in the pain evaluation algorithm. According to the results shown in Fig. 4, the decay coefficient of 0.01 best fit the decay function used in the pain evaluation algorithm, which resulted in a highest value of 0.490 for the Pearson’s correlation coefficient between VAS and PRD. It showed a positive but not perfect linear relationship between VAS and PRD index. However, it was significantly better than the result of our previous study using a novel fuzzy pain demand index (FPD) which calculated Pearson’s correlation coefficient between VAS and FPD to be 0.0098 at rest pain and 0.0027 at most pain.

Figure 4. The Pearson’s correlation coefficient between VAS and PRD against different decay coefficients used in numerical simulations of pain feeling.

Furthermore, to check the statistical relationship between PRD and VAS, the values of PRD index at each corresponding time point of VAS were compared with the original values of VAS. Therefore, the distribution of PRD index against its corresponding VAS is shown in Fig. 5(a). The values of PRD index with the same corresponding VAS scale are sorted to the same group to derive the mean and standard deviation of PRD index (shown in Table 1) for each VAS scale. Figure 5(b) shows the means and standard deviations of PRD index against their corresponding VAS scales.

In our i-pain database, only 5 records for VAS_9 and no record for VAS_10 showed that just a few people scored the VAS equal or above 9. It is possible that those patients experienced more intense nociceptive stimuli than others or interpreted the VAS scale differently from others. Moreover, the numbers of records for VAS_9 and VAS_10 were not sufficient to present the statistical results for these two treatments. Also, the mean pain scale for the 5 records of VAS_9 is smaller than the mean pain scale for those of VAS_8. Therefore, one-way analysis of variance (ANOVA) was applied to confirm the statistical correlation between two parameters (i.e., VAS and PRD index) for those records of treatments from VAS_0 to VAS_8. The analysis results of ANOVA show the statistical evidence that the PRD index significantly depended on the VAS, with $F = 125.15$ and $P$-value < 0.001. It is well known that, to examine the relationship between two parameters graphically is always a good idea (Montgomery, 2001). Figure 5(b) presents the mean-standard deviation plots for PRD index at each level of VAS and Table 1 presents statistical result between PRD index and VAS. Both of them indicate that the average of PRD index increased as VAS increased.

Therefore, a linear statistical model was used to fit the relationship between VAS and PRD index. In Table 1, the standard deviations of PRD index for all levels of VAS are close to the constant of 1.75 and it shows the variability of PRD index close to a random error. So, the correlation between PRD index and VAS can be fitted by a linear model and expressed as 

$$VAS = 2.221 \times PRD - 6.992.$$
In addition, we traced the development of postoperative pain intensity to 470 gynecological patients. Figure 6 shows the 48-hour developments of postoperative pain intensity of 470 study cases. Data was sorted by the machine-on time of PCA, and the PRD index is shown with various colors. Time is shown in day and hour of 24-hour representation. Day 1 means the day of surgical operation. Day 2 represents the next day of surgical operation.

![Figure 6. The 48-hour PRD index distributions for 470 postoperative patients of gynecological surgery. Time is shown in the practical operative time, and the pain scale is shown in various colors. The PRD index distributions were re-sorted by the time point for the occurrence of the first pain-relief demand.](image)

Furthermore, the development pattern of pain intensity for the postoperative patients of gynecological surgeries was established by averaging the first 48-hour developments of postoperative pain intensity of 470 study cases. Figure 7 shows the averaged development pattern of postoperative pain intensity of 470 gynecological patients. According to the averaged development pattern of postoperative pain intensity, pain intensity was gradually increasing during the first 10 hours after surgical operation and reached the most severe pain at the 10\textsuperscript{th} hour after surgical operation. After that, pain intensity decreased with time. The development pattern of pain intensity helps us to understand the development process of postoperative pain and facilitate better postoperative pain management.

![Figure 7. The averaged PRD index for the first 48 hours after surgical operation.](image)

Additionally, the fluctuating time series of pain feeling was extracted from the simulated time series of pain feeling by EMD, and the PRD index only depended on the first IMF (i.e., the highest-order fluctuation of pain feeling). This is because EMD and Hilbert transfer are suitable for decomposing a non-stationary signal and transferring a non-stationary time series to an analytic signal. However, neither EMD nor Hilbert transfer is not the only algorithm to solve these problems, but they are both suitable. The most critical point of this investigation was how to interpret the occurrence frequency of pain-relief demands from the original occurrence time series of pain-relief demands to the time series of PRD index. Here, we proposed an algorithm to deal with this problem, but we are still developing a better algorithm to improve it in the future.

In addition, the pharmacological effects of analgesic drugs are complicated and hard to be evaluated in clinical practice. Moreover, the pharmacological reaction to analgesic drugs depending on the physiological characteristics of human bodies is also another difficulty in pain evaluation. Thus, we simplified the complicated task of pain evaluation to a problem with the single parameter of the occurrence frequency of pain-relief demands. Nevertheless, both the characteristics of pharmacological dynamics due to various analgesic drugs and individual physiological characteristics of human bodies are the two ignored factors which result in the random errors in pain evaluation. Moreover, patients push the button for a pain-relief demand not only because their pain feeling exceeds a threshold, but also because they anticipated pain, such as changing physical position, a undergo examination, or going to sleep.

It is as well known, pain is a totally subjective feeling. It is impossible to determine a transfer equation between the PRD index and VAS, which is suitable for various objects. In this investigation, the relationship between PRD index and VAS had been determined using a linear statistical model. Totally, 2466 records of VAS scattered on the range from VAS\textsubscript{0} to VAS\textsubscript{9} were sifted from 470 study cases in our i-pain database. Both the VAS and the pain-relief demand are subjective feeling and affected by social, environmental, and psychological situation. Our presenting pain evaluation model offers a scaling algorithm for postoperative pain intensity via the occurrences of pain-relief demands based on the innovative...
signal processing algorithm of EMD. So far, it has generated an innovative continuous index of PRD, which has a highest value of Pearson’s correlation coefficient with VAS.

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References


