A Combination Method of Artificial Potential Field and Improved Conjugate Gradient for Trajectory Planning for Needle Insertion into Soft Tissue

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Abstract

To reduce postoperative complications and unwanted side-effects, injury to vital tissues and/or nerves should be avoided in minimally invasive surgery. In this paper, a combined trajectory planning algorithm that uses an artificial potential field (APF) method in conjunction with an improved conjugate gradient algorithm (ICGA) is proposed. Fascial tissues create local minima (LM) when the APF method is applied, which may lead to needle oscillation. The proposed method finds sub-goal points using ICGA to escape from LM. Thus, a complete smooth trajectory passing through anatomical obstacles is obtained. Needle guidance and steering can be performed through a unicycle model. The optimal APF function is obtained by adjusting parameter n. When n=6, the distance between LM and the target reached its minimum (1.0198), which shows that Point 6 is closest to the target and n=6 performs higher search efficiency. The experimental simulation shows combination method has higher efficiency (13.45% higher) compared to APF method, which requires just a few tenths of a second on a standard PC.

Keywords: Needle insertion, Trajectory planning, Artificial potential field, Local minimum, Improved conjugate gradient

1. Introduction

With the rapid development in robotics technology [1,2] in recent years, the use of robotic devices has been growing in surgical applications. Surgical robots have been successfully applied to a number of orthopedic applications, endoscopic surgeries such as colonoscopy [3] and laparoscopy [4], neurosurgical procedures [5,6], microsurgery [7], and even minimally invasive surgical procedures [8]. Nevertheless, even the most experienced surgical teams may still experience difficulty in finding the best trajectory for needle insertion [9]. In brachytherapy surgery, a key issue is deciding how to insert a needle while avoiding vital tissues and planting the seed accurately.

Common trajectory planning algorithms can be divided into three categories: numerical methods, inverse solution methods and search methods. Numerical methods are widely used and quickly produce an optimization routine that finds a locally optimal path, requiring just a few seconds of computation time [10,11]. A constant-time motion planning algorithm for steerable needles based on explicit geometric inverse kinematics has been proposed [12]. Search methods such as rapidly exploring random tree (RRT) and artificial potential field (APF) [13,14] compute motion plans for steerable needles to reach targets quickly.

Even though these methods improve trajectory planning, the disadvantage of these methods should not be neglected. For example, numerical methods are not applicable for all path forms. Inverse algorithms cannot guarantee solvability and search methods cannot guarantee optimal solutions. Local minima (LM) would exist when search is conducted using APF [15]. However, the APF method reduces the computation time and significantly improves search efficiency. For robot-assisted prostate brachytherapy, this study proposes a trajectory planning algorithm that uses APF in conjunction with an improved conjugate gradient algorithm (ICGA).

2. Materials and methods

2.1 Image guided robot-assisted surgical system

An ultrasound-image-guided robot-assisted surgical system is composed of three modules: preoperative planning, intraoperative navigation, and robot-assisted surgery (Fig. 1).
In preoperative planning, surgeons obtain magnetic resonance (MR) data from the patient. They import the data into a treatment planning system (TPS) to do some preoperative planning (three-dimensional (3D) reconstruction and dose planning) and the results are used in intraoperative navigation. Medical staff can use image fusion technology to fuse the real-time ultrasound image and the 3D pelvic model reconstructed from the MR image. They complete the prostate brachytherapy surgery with the help of a robot control system [16] and advanced image technology.

Figure 1. Ultrasound-image-guided robot-assisted surgical system flow chart.

2.2 Anatomical prerequisites

To obtain an optimal trajectory for needle insertion, the structures surrounding the prostate must be precisely described. Wu et al. [17] created a 3D digitized visible model of the prostate and its adjacent structures using computed tomography images. MR images are used in the present study due to their high resolution.

In humans, the prostate is a chestnut-shaped organ approximately 60 ml in volume, with maximum anterior-posterior, left-right, and superior-inferior widths of 3, 5, and 4 cm, respectively. There are several structures in direct contact with the prostate. (1) The urethra penetrates into the prostate. (2) The levator ani inferiorly supports the prostate and connects the lower part of the prostate by muscle fascia, which forms a curved surface [18]. (3) The obturator internus is positioned around the prostate and bladder [19] (Fig. 2). (4) Nerves are scattered throughout the prostate, and damage of the spiral nerve may cause some postoperative complications and unwanted side-effects. The MR imaging data in Fig. 2 were acquired using a SIEMENS 1.5-T nuclear MR imaging system.

2.3 Artificial potential field method

In medical instrument trajectory planning, the puncture needle always moves from a high potential point to a low potential point and the needle motion is in the minimum potential direction. Provided that the position of the needle is in 3D space $\mathbb{R}^3$, and the origin, $O$, is located in the lower-left quarter of the space (Fig. 3). The goal is the point of lowest potential. Gravitational potential field $U_{\text{at}}$ is defined as [20]:

$$U_{\text{at}}(p) = \frac{1}{2} k_{\text{at}} \rho^2_{\text{goal}}(p)$$  (1)

where $k_{\text{at}}$ is a gravitational scaling factor and $p \in \mathbb{R}^3$. The configuration $p$ includes a heading component and the heading angle between the puncture needle and the goal. $\rho_{\text{goal}}(p)$ is the distance between needle tip and the goal. The attractive force can be described as:

$$F_{\text{at}}(p) = -\nabla U_{\text{at}}(p) = -k_{\text{at}}(\hat{p} - \hat{p}_{\text{goal}})$$  (2)

Similarly, the obstacle is the point of highest potential. The repulsive potential field is given by:

$$U_{\text{rep}}(p) = \begin{cases} \frac{1}{2} k_{\text{rep}} \left( \frac{1}{\rho_{\text{obs}}(p)} - \frac{1}{\rho_0} \right) \rho^{n}_{\text{goal}}(p) & \rho_{\text{obs}}(p) \leq \rho_0 \\ 0 & \rho_{\text{obs}}(p) > \rho_0 \end{cases}$$  (3)

where $k_{\text{rep}}$ is a repulsive scaling factor and $\rho_{\text{obs}}(p)$ is the distance between needle tip and the obstacle. The impact factor of the obstacle $\rho_0$ defines a maximum region of repulsion and is selected based on the size of each obstacle. The repulsive force can be described as:

$$F_{\text{rep}}(p) = -\nabla U_{\text{rep}}(p) = \begin{cases} F_{\text{rep},1}(p) + F_{\text{rep},2}(p) & \rho_{\text{obs}}(p) \leq \rho_0 \\ 0 & \rho_{\text{obs}}(p) > \rho_0 \end{cases}$$  (4)

$$F_{\text{rep},1}(p) = k_{\text{rep}} \left( \frac{1}{\rho(p)} - \frac{1}{\rho_0} \right) \rho^n_{\text{goal}}(p) \rho^2(p)$$  (5)

$$F_{\text{rep},2}(p) = k_{\text{rep}} \left( \frac{1}{\rho(p)} - \frac{1}{\rho_0} \right) \rho^2_{\text{goal}}(p)$$

Figure 2. Physiological anatomical schematic diagram of area surrounding prostate. (a) Transverse schematic of the fascia. It includes various structures surrounding the prostate based on a MR image where the colors yellow, light blue signify obturator internus fascia and levator ani fascia, respectively. (b) 3D reconstruction environment schematic with obstacles and the target, where the tumor in prostate is considered as a target while the urethra is considered as an obstacle.

Figure 3. Schematic diagram of space coordinates.
\[ \mathbf{F}_{rep}(p) = \frac{n}{2} k_{rep} \left( \frac{1}{\rho_0(p)} - \frac{1}{\rho_0} \right)^2 \rho_0^{-n} g_{rep}(p) \]  

(6)

The scaling factors \( k_{at} \) in Eq. (1) and \( k_{rep} \) in Eq. (3) are chosen such that the attraction and repulsion potentials are of the same order of magnitude. The two scaling factors were viewed as empirical parameters in Simon’s study [14] with the range of 0–1, \( n \) is a parameter to be optimized for planning a more reasonable trajectory.

The potential field and resultant force are respectively:

\[ \mathbf{U}(p) = \mathbf{U}_{at}(p) + \mathbf{U}_{rep}(p) \]  

(7)

\[ \mathbf{F}(p) = \mathbf{F}_{at}(p) + \mathbf{F}_{rep}(p) \]  

(8)

It is assumed that the soft tissue does not create deformation during needle insertion. A spatial discretization method is used to simulate the piercing process of needle insertion. As shown in Fig. 4(a), a compartment is composed of four cubic elements around the first insertion point \( s \). The size of each compartment is determined by the insertion distance at each step of the insertion process. The distance between nodes is 0.25 mm. To determine the direction of the next step in the insertion process, four cubic elements surrounding \( s \) are considered. The initial position is denoted by \( s \), and \( s_1 \) are the surrounding nodes. The potential value \( \mathbf{G}(i) \) at each node is computed to select the node with the minimum potential.

\[ \mathbf{G}(i) = \mathbf{G}_{at}(i) + \mathbf{G}_{rep}(i) \]  

(9)

where \( \mathbf{G}_{at}(i) = \mathbf{U}_{at}(i) \) and \( \mathbf{G}_{rep}(i) = \mathbf{U}_{rep}(i) \)

### 2.4 Improved conjugate gradient algorithm

The control of needle insertion must be considered in brachytherapy. The direction angle \( \alpha \) selected by ICGA cannot exceed the maximum deflection angle \( \theta \) of needle insertion. Therefore, when \( \alpha > \theta \), \( \theta \) is selected as the search direction angle, and when \( \alpha < \theta \), \( \alpha \) is selected as the search direction angle (Fig. 4(b)). Suppose that the puncture needle is located at point \( p_{mn} \) when it is stuck in an LM. Then, the search direction in the first step can be described as:

\[ \mathbf{s}^{(1)} = -\nabla U(p^{(1)}) = -\mathbf{s}_I \]  

(10)

where \( p^{(1)} \) is the initial search point and \( \mathbf{s}^{(1)} \) is the initial search direction.

The search proceeds from point \( p^{(k)} \) along its negative gradient direction until \( p^{(k+1)} \) is found. The conjugate direction obtained in the previous step is used as the search direction in next step:

\[ \mathbf{s}^{(k+1)} = -\nabla U(p^{(k+1)}) + \beta \mathbf{s}^{(k)} \]  

(11)

where, \( p^{(k+1)} \) is the point found by ICGA. \( \mathbf{s}^{(k+1)} \) is the subsequent search direction. The value of \( \beta \) should make the two non-zero vectors \( \mathbf{s}^{(k)} \) and \( \mathbf{s}^{(k+1)} \) be conjugate. That is:

\[ \mathbf{s}^{T} \mathbf{s} = 0 \quad (k = 1, 2, 3, ..., n) \]  

(12)

The iterative formula is:

\[ p^{(k+1)} = \mathbf{p}^{(k)} + \chi \mathbf{s}^{(k)} \]  

(13)

\[ g_k = \nabla U(p^{(k)}) = b + \mathbf{A} p^{(k)} \]  

(14)

\[ g_{k+1} = \nabla U(p^{(k+1)}) = b + \mathbf{A} p^{(k+1)} \]  

(15)

\[ g_{k+1} | g_k | g_s = 0 \quad (k = 0, 1, 2, ..., n) \]

(16)

Then, \( \mathbf{s} \) can be derived and applied to obtain the search direction in the next step:

\[ \beta = \frac{g_k^T g_{k+1}}{g_k^T g_k} \]  

(17)

The convergence check equation is:

\[ \| \nabla U(p^{(k+1)}) \|^2 < \epsilon = \cos \theta \]  

(18)

A one-dimensional search is started, and the optimal step length \( \chi^{(k)} \) and new point \( p^{(k+1)} \) are computed (Eq. 13):

\[ \min U(p^{(k)} + \chi \mathbf{s}^{(k)}) = \mathbf{s}^{(k)} \]  

(19)

The final sub-goal point \( s_i \) is selected as a virtual target. Moreover, \( \mathbf{G} \) and \( \mathbf{O} \) are not considered at this moment. Only the gravitational potential energy of sub-goal point \( s_i \) is considered. The potential function is reset as:

\[ \mathbf{U}(p) = \mathbf{U}_{at}(p) = \frac{1}{2} k_{at} p^2 \]  

(20)

The search algorithm that combines APF and ICGA is shown in Fig. 5.

### 2.5 Unicycle model

Consider a bevel-tip needle driven with two velocity inputs, namely insertion speed and rotation speed, actuated from the base of the needle. As the needle is inserted into tissue, the tissue imposes a reaction force on the bevel that deflects the needle tip, causing it to follow an arc. The rotational input at the base causes the needle to turn about its shaft, reorienting the bevel. In the unicycle model, the insertion speed and rotation speed are viewed as inputs to a kinematic nonholonomic system. Roughly speaking, inserting the needle at speed \( v_i \) is like riding a bicycle along a circular arc of radius \( r \) while rotating the needle at speed \( v_r \) reorients the plane containing the bicycle’s path (Fig. 6). During penetration, the equation of the locus can be expressed as:
3. Results and discussion

3.1 Preliminary trajectory planning results for needle insertion into different tissues

As mentioned in Section 1.2, the urethra, nerves, and fascia are vital tissues whose puncture should be avoided during prostate brachytherapy. Based on anatomical prerequisites and the shape of some vital tissues, a bent pipe, a spiral pipe, and a curved surface model are used to simulate the urethra, nerves, and fascia, respectively (Fig. 7). Suppose that the initial position of the puncture needle is (3, -1, 0) cm. Figure 7 shows the trajectory for needle insertion using APF.

![Figure 7. 3D trajectory planning result. G, O and T stand for the goal, obstacle, and the trajectory, respectively. In (c), T1, T2 and T3 represent three different trajectories.](image)

Figures 7(a) and 7(b) show that the needle bypasses the bent and spiral pipe models to reach the target. Nevertheless, the trajectory terminates in front of the curved surface model (Fig. 7(c)). The initial position of the puncture needle was thus changed. Figure 7(d) shows for initial needle position of (3, -1, 0), (2.7, -1, 0), and (2.4, -1, 0) cm, the needle still vibrates at \( p_{\text{min}} \) persistently and cannot move out of the scope, which is called an unavoidable LM problem.

3.2 Simulation of escaping from local minimum

Suppose that the target is at position (2.2, 17.1, 5) cm and the initial position of the needle is (2.5, -1, 0) cm. Paths \( l_1, l_2, \) and \( l_3 \) are the three phases of the trajectory, respectively. Path \( l_3 \), which is between two blue dots, represents the trajectory escaping from LM position \( p_{\text{min}} \). \( \ell_1 \) is the sub-goal point and \( p_1, p_2, p_3 \) and \( p_4 \) are inflection points in the search trajectory (Fig. 8(a)). Moreover, the trajectory was optimized using cubic spline interpolation (Fig. 8(b)). The computation speed values are summarized in Table 1.
The computation time of trajectory planning is transient, especially in the environment with bent pipe and spiral pipe obstacles. Only a few tenths of a second (mean value of 0.9061s) are required to find the trajectory. In the environment with a curved surface obstacle, more time (mean value of 1.0470s) is required to find a feasible path due to the existence of an LM point. The proposed combination method uses less time than APF method when curved surface model is used. The search algorithm becomes ICGA as soon as the needle tip is stuck in the LM area. The proposed combination method improves search efficiency to 13.45%. The algorithm does not waste a lot of time in the LM area, and is thus applicable for a real-time simulation or training system.

3.3 Optimization of APF function and simulation in complex environment

As shown in Fig. 8(c), position of the LM is influenced by value of $n$. The goal is at location (2.2,1.7,1.5) cm. The 3D coordinates of the nine points are shown in Table 2. The experimental data indicate that the minimum value (1.0198cm) is reached when $n = 6$, which shows that Point 6 is closest to the goal relative to other points. Similarly, the total number of search iterations decreases until $n = 6$. That is, when $n = 6$, the search path is shortest from the LM to the target and the total number of search iterations is least, giving the shortest search time and improving the efficiency of the search algorithm. The value of $n$ can be adjusted to acquire the best potential field function.

### Table 1. Computation time values for trajectory search.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Artificial potential field</th>
<th>Combination method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4109</td>
<td>0.3086</td>
</tr>
<tr>
<td>2</td>
<td>0.4192</td>
<td>0.3037</td>
</tr>
<tr>
<td>3</td>
<td>0.4159</td>
<td>0.3315</td>
</tr>
<tr>
<td>4</td>
<td>0.4216</td>
<td>0.3132</td>
</tr>
<tr>
<td>5</td>
<td>0.4196</td>
<td>0.3140</td>
</tr>
</tbody>
</table>

The bent pipe, spiral pipe, and curved surface models were placed in 3D space and the target was placed in this complex environment. The results of trajectory planning are shown in Fig. 8(d).

### Table 2. Distance to goal and number of iterations for various local minimum points.

<table>
<thead>
<tr>
<th>Value of $n$</th>
<th>3D coordinates of $n$</th>
<th>Distance to goal (cm)</th>
<th>Total number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2.2, 0.0, 1.0)</td>
<td>1.6553</td>
<td>366</td>
</tr>
<tr>
<td>2</td>
<td>(2.2, 0.13, 1.13)</td>
<td>1.4838</td>
<td>347</td>
</tr>
<tr>
<td>3</td>
<td>(2.2, 0.2, 1.24)</td>
<td>1.3790</td>
<td>323</td>
</tr>
<tr>
<td>4</td>
<td>(2.2, 0.3, 1.3)</td>
<td>1.2650</td>
<td>310</td>
</tr>
<tr>
<td>5</td>
<td>(2.2, 0.44, 1.44)</td>
<td>1.0914</td>
<td>297</td>
</tr>
<tr>
<td>6</td>
<td>(2.2, 0.5, 1.5)</td>
<td>1.0198</td>
<td>283</td>
</tr>
<tr>
<td>7</td>
<td>(2.3, 0.5, 1.5)</td>
<td>1.0247</td>
<td>289</td>
</tr>
<tr>
<td>8</td>
<td>(2.36, 0.47,1.49)</td>
<td>1.0633</td>
<td>295</td>
</tr>
<tr>
<td>9</td>
<td>(2.5, 0.45, 1.48)</td>
<td>1.1139</td>
<td>306</td>
</tr>
</tbody>
</table>

4. Conclusion

A trajectory planning algorithm of needle insertion into soft tissue based on the 3D APF method in conjunction with ICGA was proposed. The LM problem in a 3D environment with fascia when using APF was solved using ICGA, which finds sub-goal points to escape from an LM. The optimal potential field function is defined by adjusting the value of parameter $n$. The proposed combination method improves search efficiency to 13.45%. In the future, the properties of soft tissue will be taken into consideration, and a dynamic trajectory planning method will be studied [21]. The relationship between needle deflection and tissue deformation will be determined and applied to trajectory planning.

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