Fast Statistical Image Reconstruction for Emission Tomography: Application to SPECT

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Abstract

Nuclear medicine imaging, including PET and SPECT, has become a powerful diagnostic tool, in that nuclear medicine imaging is able to visualize physiological functions or functional metabolism. In nuclear medicine, an estimate of the three-dimensional spatial distribution of injected radionuclide is needed for diagnosis. This estimate is obtained through a reconstruction algorithm. Statistical image reconstructions have been proven to outperform the traditional FBP reconstruction in many aspects. However, the main problem with statistical reconstruction is its computation load and slow convergence. The OSEM (ordered subset expectation maximization) algorithm, which has an order-of-magnitude speed enhancement over the original MLEM (maximum likelihood EM) algorithm, is the most popular statistical reconstruction method used in many clinical hospitals worldwide. Nevertheless, the OSEM algorithm does not converge for real data so that its noise and resolution are difficult to predict. Furthermore, the OSEM is not easy to include a prior term. Many fast OS-type algorithms were proposed to solve for the speed and the convergence problems of the OSEM. They all need a user-specified relaxation schedule to ensure the speed and convergence property. But there is no easy way to decide the relaxation schedule. Previously, we had proposed a fast, convergent OS-type algorithm, called COSEM-MAP, to retain the speed while at the same time without any relaxation schedule. Here, we briefly review the COSEM algorithm, and then apply the method to a set of real SPECT phantom data acquired from the Chang Gung Memorial Hospital. A segmented attenuation correction method is applied to the SPECT data. The COSEM-MAP result is compared to those of the popular FBP and OSEM methods often used in clinical reconstruction. Our COSEM-MAP method shows potential effectiveness as compared to the FBP and OSEM.

Keywords: SPECT, Statistical Reconstruction Method, Iterative Reconstruction, PET

Introduction

Emission tomography in nuclear medicine, including PET (positron emission tomography) and SPECT (single photon emission computed tomography), provides a useful diagnostic tool for investigating functions and physiology in the human body in-vivo. The way that emission tomography works is to inject radioactive tracer into a patient's body, and the tracer travels to an organ or tissue of interest through metabolic functions. When it resides in the specific tissues, it decays and emits gamma rays (photons) and the amount of emitted photons depends on the physiology of the tissues. Compared to normal cases, tissues with diseases often emit more or less photons. Therefore, we can tell the abnormal tissues by investigating the photon-emitting rate for each tissue. The photon-emitting rates (object) can be “reconstructed” from collecting photons (data) emitted from the body, and the data is collected by a special gamma camera around the patient's body.

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The collected data contains the photons from the overlapped individual tissues along a collecting volume (ideally, a ray), and the cross-sectional information has been lost in the data. To obtain a “tomographic” image for one cross-section of the body, one then needs a reconstruction. A reconstruction is a mathematical operation by ‘converting’ the data back into its associated object. Therefore, it is important to have a reconstruction as accurate as possible to allow more accurate diagnosis and quantitation. One can roughly divide the methods of reconstruction into two groups, analytical methods like FBP (filtered back-projection) reconstruction, and statistical methods like ML (maximum likelihood) and MAP (maximum a posteriori) reconstructions. FBP is the most popular reconstruction method in the field of nuclear medicine due to its speed property. Statistical reconstructions, however, have outperformed the FBP reconstruction with the abilities of photon noise handling and physical effects modeling. The difference becomes significant in low-count situations. Unfortunately, the drawback of a statistical reconstruction is its slow convergence.

A statistical reconstruction often contains the following
elements: an objective function, and an optimization algorithm. Since photon noise follows Poisson distribution, the Poisson likelihood is naturally a good choice for the objective function. Rockmore and Macovski [1] first introduced the ML principle on the Poisson likelihood for image reconstruction. However, without a feasible algorithm for computing ML estimates, the ML method still remained impractical for image reconstruction problems with thousands of parameters to estimate. Later, the difficult of computing ML estimates was overcome by the introduction of the iterative EM (expectation-maximization) algorithm to emission tomography [2]. The advantage of EM-ML reconstruction is its easy programming and positivity-enforcement. However, it still suffers from slow convergence.

In addition to the speed problem, due to the ill-conditioning of the image reconstruction problem, the noise in an image reconstructed using ML starting to grow with iteration as the reconstruction proceeds to success converge. One can reduce the noise by using stopping criteria to terminate ML before image quality deteriorates. Alternatively, one can alleviate the instability of the ML reconstruction by using Bayesian methods, which are equivalent to penalized maximum likelihood methods for the case of maximum a posteriori (MAP) reconstruction.

To make the statistical reconstruction methods clinically useful, the speed problems in statistical reconstructions have drawn much attention recently. Among all, an ordered subsets EM (OS-EM) algorithm proposed by Hudson et al. [3], has been used extensively in emission reconstruction and inspired many OS-type reconstruction methods. OSEM algorithm uses only a subset of the projection data per backprojection. The popularity on the use of OS-EM is primarily because of its order-of-magnitude acceleration over EM-ML, and easy implementation with only a slight modification of the existing EM algorithm. However, the OS-EM has problems of lack of convergence and limit cycles [3].

In a parallel development, a row-action maximum likelihood algorithm (RAMLA) [4] was proposed by including strong under relaxation in a modified version of OSEM. Later [5], the authors extended their approach to the MAP case, termed BSREM. Similar development by [6], they also proposed convergent OS-type algorithms called OS-SPS, and modified BSREM. The convergent RAMLA, BSREM and OSSPS all need a relaxation schedule to ensure the convergence, and the main problem is that the selection of relaxation schedule is not easy while simultaneously satisfying the convergence requirement.

Previously, we had derived a convergent OS-type ML reconstruction, COSEM-ML (Complete data OSEM-ML) [7], and extended to a MAP method called (COSEM-MAP) [8, 9]. The advantage of our proposed algorithms is that it is fast, and convergent [9] without any relaxation schedule needed as that in BSREM [5] and OSSPS [6]. From the simulation, we have proved that our proposed COSEM reconstruction is faster than the EM-ML while is nearly as fast as BSREM. Here, we apply our COSEM-MAP algorithm to real SPECT data acquired in a clinical protocol, and compared that to the result of FBP, and OSEM reconstructions, both are often used in the clinical situation. Our result showed that our COSEM-MAP reconstruction produced better result than that of FBP and similar performance as that of OSEM in terms of contrast recovery.

Below, in Sec.II, we briefly review our previously proposed COSEM-MAP algorithm, as well as the EM-ML and OSEM algorithms. In Sec. III, we provide the comparison result of COSEM-MAP, OSEM, and FBP using real data acquired in SPECT. The discussion is included in Sec.IV.

Method

The object (the mean emission rate) is expressed by a vector $f$ with elements $\{ f_m; m=1,..,N \}$, and the projection data $g=\{ g_m; m=1,..,M \}$. We denote an estimate of $f$ as $\hat{f}$, and the projection data $g$, which has Poisson distribution with mean equal to $g = HF$. Here $H$ is the system matrix with element $H_{mn}$ indicating the probability of counts from pixel $m$ received by bin $n$ $(m=1,..,M; n=1,..,N)$. The Poisson loglikelihood is then:

$$\Phi_f (g; f) = \sum_m \left\{ g_m \log \frac{g_m}{f_m} - g_m \right\}$$

(1)

where the mean is $\bar{g}_m = \sum_n H_{mn} f_n$.

**EM-ML**

One can optimize the loglikelihood (1) by any suitable algorithm. However, due to the coupling of the object pixels in the objective function, direct optimization of (1) leads to an equation that is not solvable explicitly. One way to solve the optimization difficulty is to apply the EM algorithm [2] by introducing a set of 'complete' data, which will make the estimation problem easier. The EM algorithm is formulated through a statistical frame, and contains the E (expectation) and M (maximization) steps [2]. Previously, we had introduced a re-derivation of the EM algorithm through a new objective function, and that is not based on a statistical framework.

It turns out that the EM-ML update equations can be re-derived by the following "complete-data" objective function [7][8]:

$$E_{ML}(C; f) = \sum_m \sum_n \phi(C_m; H_{mn} f_n) + \sum_m \sum_n C_m - g_m$$

(2)

where $\phi(x, y) = x \log \frac{x}{y} - x + y$ is an 1-Divergence function [10], and the complete data $C (\{ C_m; m=1,..,Mn=1,..,N \})$, indicates the number of counts collected in bin $m$ emitting from pixel $n$. Note that $\{ \tilde{C}_m; m=1,..,M \}$ are the Lagrange parameters that express the complete-incomplete data constraint

$$\sum_n C_m = g_m$$

The alternating minimization on $C$ and $f$ is performed by first minimizing Eq.(2) with respect to $C$ and $\tilde{C}_m$ while keeping $f$ fixed [7][8], and this leads to the $C$ update:

$$\tilde{C}_m^{k+1} = g_m \frac{H_{mn} f_n^{k}}{\sum_j H_{mn} f_j^{k}}, \forall m, n$$

(3)

This update (3) is exactly the E-step of the standard EM-ML algorithm. Next we optimize $E_{ML}(C; f)$ w.r.t. $f$ with fixed $C$, and
this produces the $\mathbf{f}$ update:

$$j_{n}^{k+1} = \frac{\sum_{m} \hat{f}_{m}^{k+1}}{\sum_{m} H_{mn}}, \forall n.$$  \hspace{1cm} (4)

Note that equations (3) and (4) are the result of an alternating coordinate descent algorithm, and are identical to the EM-ML algorithms [2]. The EM-ML reconstruction (3)(4) will generate positive reconstruction if provided positive initial estimate due to the multiplicative property.

**OSEM**

However, EM-ML converges slowly. The OSEM [3] modified the EM-ML update schedule (4) by first dividing the projection data $[1,...,M]$ into $L$ disjoint subsets $\{S_{l}, l=1,...,L\}$, and replacing the full data set summation by only a subset summation each time. Thus, the resulted update equation at $(k, l)$-th iteration becomes:

$$j_{n}^{k(l)} = \frac{\sum_{m} \sum_{S_{l}} H_{mn} g_{m}}{\sum_{m} H_{mn} g_{m}^{(k(l)-1)}} \hspace{1cm} \forall n, \ l = 1,..,L$$  \hspace{1cm} (5)

for $n=1,..,N$, and where $S_{l}$ indicates the $l$-th subset projection data for $l=1,..,L$. Note that for each iteration $k$, there are $L$ sub-iterations, and each update $j_{n}^{k(l)}$ at sub-iteration $l$ uses only a subset of the projection, and serves as the initial estimate for next sub-iteration $j_{n}^{k(l+1)}$. Also $j_{n}^{k(l)} = f_{n}^{k+1}$. That is, for each iteration $k$, there are $L$ sub-iterations. Note that the OSEM (5) has order-of-magnitude speed advantage over the EM-ML (3), but has the problem of limit cycles, and no theoretical convergence proof.

**COSEM-ML**

From equations (3) and (4), the COSEM-ML algorithm [7] can be described as follows: Similar to that in OSEM, we can apply the idea of ordered subsets $\{S_{l}, \ 1,...,L\}$. Rather than using the ordered subsets on the projection data $\mathbf{g}$, we now apply the subsets on the complete data $\mathbf{C}$. First, we can rewrite the objective function (2) as:

$$E(\mathbf{C}; \mathbf{f}) = \frac{1}{l} \sum_{i} \sum_{l} \sum_{m} \left[ f_{mn} \log f_{mn} + \frac{1}{l} \sum_{l} \sum_{m} \sum_{n} \left[ C_{mn} - g_{m} \right]^{2} \right].$$  \hspace{1cm} (6)

Now, let’s optimize Eq.(6) with two loops, the outer loop indexed by $k$, and the inner loop by $l$. At iteration $k$ and the inner loop sub-iteration $l$, Eq.(6) is first minimized w.r.t. $C_{mn}$ for a given subset $m \in S_{l}$ while keeping $\{C_{mn}, m \notin S_{l}\}$ and $\mathbf{f}$ fixed, and the optimization leads to the following update equation,

$$C_{mn}^{(k,l)} = g_{m} \sum_{l} \sum_{m} H_{mn} j_{n}^{(k,l)} \hspace{1cm} \forall n, m \in S_{l}$$  \hspace{1cm} (7)

Next, let’s keep $C_{mn}$ fixed (for all $m, n$), and optimize (6) w.r.t. $\mathbf{f}$. However, instead of using only the $i$th subset $\{m \in S_{i}\}$, all data $\{m=1,...,M\}$ are used to get the update equation for $\mathbf{f}$:

$$j_{n}^{(k,l)} = \frac{1}{l} \sum_{l} \sum_{m} C_{mn}^{(k,l)} + \frac{1}{l} \sum_{l} \sum_{m} \sum_{n} \sum_{m} H_{mn} j_{n}^{(k,l)} \hspace{1cm} \forall n, l = 1,..,L$$  \hspace{1cm} (8)

This COSEM update (8) is different from that of OSEM in that, only a subset of data points are used in the OSEM update equation (5) while all data points are used in the COSEM update equation (8). But at the same time, only a subset of the complete data $\mathbf{C}$ is updated in both OSEM and COSEM.

**COSEM-MAP**

To consider a MAP reconstruction, one can simply add a prior term into the Poisson objective function (1) to form a MAP objective function. Similar to that in ML, the MAP solution can be obtained by optimizing the following complete data MAP objective [8][9]:

$$E_{MAP}(\mathbf{C}; \mathbf{f}) = E_{ML}(\mathbf{C}; \mathbf{f}) + \lambda E_{P}(\mathbf{f})$$  \hspace{1cm} (9)

where $E_{P}(\mathbf{f})$ is the prior term, and $\lambda$ the smoothing parameter. Here we use the prior term,

$$E_{P}(\mathbf{f}) = \sum_{n} w_{mn} \left[ f_{n} - f_{d} \right]^{2}.$$  \hspace{1cm} (10)

Here, the neighborhood system is indicated by $\mathcal{N}(n)$, and the weight between pixel $n$ and $n'$ is expressed by $w_{mn'}$.

By optimizing the Bayesian objective (9) w.r.t. $\mathbf{C}$ and $\mathbf{f}$ as in the case of COSEM-ML with only the likelihood function, one can obtain the final update equation. However, due to the coupling of pixels in the prior (10), it is not easy to optimize (9) w.r.t. the object $\mathbf{f}$. One way is to use the so-called OSL (one-step late) scheme [11], which uses old reconstruction at previous iteration as a decoupling procedure. However, OSL is heuristic and inaccurate. One should avoid this method.

It turns out that one can use a separable surrogate prior [12] to replace (10)

$$E_{SP}(\mathbf{f}) = \frac{1}{l} \sum_{i} \sum_{l} \sum_{m} \left[ f_{n} - f_{n}^{old} - f_{d}^{old} \right]^{2} + \left[ -2 f_{n}^{old} + f_{n}^{old} + f_{d}^{old} \right].$$  \hspace{1cm} (11)

Therefore with the new surrogate prior, the complete data MAP objective becomes

$$E_{MAP-SP}(\mathbf{C}; \mathbf{f}) = E_{ML}(\mathbf{C}; \mathbf{f}) + \lambda E_{SP}(\mathbf{f})$$  \hspace{1cm} (12)

Similar to the COSEM-ML, the COSEM-MAP solution can be obtained by an alternating optimization on the MAP objective (12). First, at iteration $(k, l)$, we optimize (12) w.r.t. $C_{mn}$ for a given subset $m \in S_{l}$ while keeping $\{C_{mn}, m \notin S_{l}\}$ and $\mathbf{f}$ fixed, and this leads to the same update equation for $C_{mn}$ in (7).

Next, we optimize (12) w.r.t. $\mathbf{f}$ while keeping others fixed, and this leads to a quadratic equation for $\mathbf{f}$ as reported in [8]:

$$f_{n}^{(k,l)} = -b + \sqrt{b^{2} + 4ac} \hspace{1cm} 2a$$  \hspace{1cm} (13)

where

$$a = 4 \lambda \sum_{n} w_{mn}^{old},$$

$$b = -2 \sum_{n} w_{mn}^{old} j_{n}^{(k,l)} + j_{n}^{(k,l)} \sum_{m} H_{mn}$$

$$c = -B_{n}^{(k,l)}.$$  \hspace{1cm} (14)

Note that for book-keeping purpose, we keep track of the value $B_{n}^{(k,l)} = \sum_{m} C_{mn}^{(k,l)}$. Thus, our COSEM-MAP algorithm [8] can be summarized in these two equations (7) and (13).
The cylindrical phantom has a diameter of 21.6 cm, and contains 6 small spheres inside the phantom. Each sphere has diameter of 31.8 mm, 25.4 mm, 19.1 mm, 15.9 mm, 12.7 mm, and 9.5 mm, separately. The background of the phantom was filled with a Tc-99m solution (22.4 Bq/mL). All spheres were filled with a higher radioactivity of the same radiotracer (89.7 Bq/mL) with ratio of 4:1 to the background at the time of image acquisition. Then we acquired two data sets of high-count data, and low-count data. The number of scanning projections is 120, and each with 128 bins, and 128 slices. For the high-count data, the scan time at each projection angle is 20 seconds, while for the low-count data 10 seconds.

Since the FBP and OSEM are the most popular reconstruction methods used in the clinical situations, here we reconstruct the two real phantoms data sets using the methods of FBP, OSEM, and our COSEM-MAP. The reconstruction matrix is of size 128x128x128 voxels. Because no transmission data is available, for the attenuation correction, we used a segmented attenuation map as used in the attenuation correction, that was obtained in the initial reconstruction to obtain a segmented attenuation map. Then we applied the SAC method for obtaining an initial reconstruction for each reconstruction from the high-count and low-count data. To compare the contrast recovery for each reconstruction, we select five ROIs from the visible 4 spheres, and the background, from this transverse slice. The sizes of the five ROIs are background: 5*5 pixel; R1: 5*5 pixel; R2: 5*5 pixel; R3: 4*4 pixel; R4: 3*3 pixel. We then measure the mean value of each ROI, and compute the ratio of ROI to background for each ROI. The result is shown in Table 1 for the high-count data, and Table 2 for the low-count data. Note that the true ratio for each ROI should be 4. From Table 1, at ROI 1, both OSEM and COSEM-MAP have higher contrast recovery than that of FBP, but lower contrast recovery than that of OSEM at ROI 2 and 3. For low-count data in Table 2, OSEM wins only at ROI 1, while COSEM-MAP has lower ratio than both FBP and OSEM at all ROIs.

**Discussion**

We had proposed a fast, convergent, OS-type algorithm, COSEM-MAP, in [8]. Here we applied the COSEM-MAP algorithm to SPECT real data to show its effectiveness, and compare with the result of FBP, and the OSEM reconstructions. Since there is no transmission data available in this SPECT acquisition, we also applied the SAC method for obtaining an attenuation map as used in the attenuation correction. We compare the contrast ratio of each hot ROI to the background for each reconstruction from the high-count and low-count acquisitions. The result showed the COSEM-MAP method performed well only for the largest ROI and the smallest ROI, and relatively worse in other cases.

The reason for the low ratio of ROIs in the COSEM-MAP reconstructions might be two-folded: the smoothing parameter, and number of iterations in the COSEM-MAP. There are some relationship between the post-smoothing and the smoothing penalty from the prior as studied in [15]. This will be a future work to study the effects of both post-smoothing, and the

**Table 1. Values of R1/BG, R2/BG, R3/BG, and R4/BG for various reconstruction methods on standard data.**

<table>
<thead>
<tr>
<th>ROI</th>
<th>FBP</th>
<th>OSEM</th>
<th>COSEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>True</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>FBP</td>
<td>3.7494</td>
<td>3.9054</td>
<td>3.6291</td>
</tr>
<tr>
<td>OSEM</td>
<td>3.9608</td>
<td>3.7172</td>
<td>3.3896</td>
</tr>
<tr>
<td>COSEM</td>
<td>3.8593</td>
<td>3.4673</td>
<td>3.0962</td>
</tr>
</tbody>
</table>

**Table 2. Values of R1/BG, R2/BG, R3/BG, and R4/BG for various reconstruction methods on low-count data.**

<table>
<thead>
<tr>
<th>ROI</th>
<th>FBP</th>
<th>OSEM</th>
<th>COSEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>True</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>FBP</td>
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<td>3.6935</td>
<td>3.2086</td>
</tr>
<tr>
<td>OSEM</td>
<td>3.9483</td>
<td>3.2988</td>
<td>2.8500</td>
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<tr>
<td>COSEM</td>
<td>3.6260</td>
<td>3.9003</td>
<td>2.7831</td>
</tr>
</tbody>
</table>
Fast Statistical Reconstruction for SPECT

Figure 2: Anecdotal reconstructions of high-count data from (a) FBP, (b) COSEM-MAP, and (c) OSEM, and low-count data from (d) FBP, (e) COSEM-MAP, and (f) OSEM. Due to the limited resolution in SPECT, only 4 spheres are visible in the reconstructions.

smoothing prior through the smoothing parameters. In addition, as expected [7][8], the speed of COSEM-MAP is slower than that of BSREM faster (convergent algorithm), and 10 iterations might not be good enough for COSEM-MAP to reach a stable reconstruction. This one can expect the ratio of ROIs in the COSEM-MAP reconstructions will increase at higher iterations. One alternative way of improving the image quality is to use an enhanced COSEM algorithm [16]. We will test this method [16] using real phantom data as a future work.

Also, we only considered the attenuation correction in the physical model for both OSEM and COSEM-MAP. For the future work, we will include the effect of detector response, and this might increase the contrast recovery ratio. We will also investigate various scan protocol of the SPECT data (i.e. different scanning angle, and scanning time) using the methods in this work.

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Reference