The Thresholding MLEM Algorithm

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Abstract

The maximum likelihood expectation maximization (MLEM) algorithm has several advantages for image reconstruction over the conventional filtered backprojection (FBP). However, the slow convergence rate and the high computation cost for a practical implementation have impeded its clinical applications. In this study, we propose the incorporation of thresholding technique into MLEM to speed up the convergence rate. Owing to the fact that the reconstruction time is proportional to the total number of pixels (voxels), the thresholding technique that nullifies the value of a pixel if it falls below a threshold, can effectively remove the non-active pixels and significantly speed up the reconstruction. Preliminary tests on simulated PET data show that the thresholding technique speeds up the convergence rate and reduce error in the reconstructed image.

Keywords: MLEM, Threshold, Iterative reconstruction

Introduction

Emission Computed Tomography (ECT) can be used to obtain image of the radioactivity distribution to extract patient’s physiological information. The projection images of the activity distribution are measured through the detector rings around the patient and are reconstructed with different reconstruction algorithms. FBP technique is the standard reconstruction algorithm. It is based on direct inversion of Radon transform derived using continuous sampling and discretized for sampled data. Because of the limited number of ECT projection sets, FBP introduces streak artifacts in the reconstructed images. Despite of this disadvantage, FBP is used extensively in nuclear medicine due to its short reconstruction time [1].

The problem of image reconstruction for ECT can be viewed as a standard statistical estimation problem. Expectation-maximization (EM) algorithm [2] is an approach to iterative computation of maximum-likelihood estimates when the observations can be viewed as incomplete data. The maximum-likelihood approach is first introduced in image reconstruction by Rockmore and Macovski [3]. The MLEM is then proposed and implemented for tomographic reconstruction [4-6]. The MLEM reconstructs images by iteratively maximizing a likelihood function. The advantages of MLEM over FBP technique are (1) equally spaced projection data are not required; (2) an incomplete set of projection data can still be utilized; and (3) the statistical noise artifact is reduced.

Major disadvantages of the MLEM reconstruction algorithms are the slow convergence rate and the high computational cost for a practical implementation. Several techniques were employed to solve these problems [7-12]. Hudson and Larkin [8] proposed the ordered subsets EM (OSEM) to group projection data into an ordered sequence of subsets. The MLEM reconstructions are calculated only for the data in the selected subset. This method iterates at a fraction of the time required by the conventional MLEM procedure, where reprojectons are performed using the whole set of data. Raheja et al [12] used multiresolution approach in both image reconstruction and detector space. The algorithm iterates for various multiresolution grid levels and projection data and is able to achieve faster convergence because of the reduced matrix used in MLEM.

In this paper, we propose to incorporate a simple thresholding technique into both MLEM algorithms. At the end of typical iteration, the estimated pixel values were compared with a threshold to nullify pixels with small activities, which are mostly noisy background. The reconstruction can be sped up because only the active pixels need to be calculated.

Methods

A. MLEM

Let \( n^*(d_i, d) = \sum \) be the measured data in detector tube \( d \). It is a Poisson distribution and can be expressed as
\[ f(n^*|d) = P(n^* = n^*|d) = e^{-\lambda^*(d)/n^*(d)!} \] (1)

where

\[ \lambda^*(d) = \sum_{b=1}^{B} \lambda(b)p(b,d) \] (2)

is the expectation of \( n^*(d) \), and \( \lambda(b) \) is the activity density in pixel \( b \) to be estimated from the measured data. The probability matrix \( p(b,d) \) represents the probability that an emission event in pixel \( b \) to be detected in detector tube \( d \).

The likelihood of the observed data is

\[ L_c(\lambda) = \prod_{d=1}^{D} e^{-\lambda^*(d)/n^*(d)!} \] (3)

If \( \lambda(\lambda) = \log L_c(\lambda) \), then it can be shown that

\[ \lambda(\frac{\partial}{\partial \lambda(b)}) = -\sum_{d=1}^{D} p(b,d) + \lambda(b) \sum_{d=1}^{D} \lambda(b)p(b,d) \] (4)

Differentiate \( \lambda(\lambda) \) with respect to \( \lambda(b) \), multiply both sides by \( \lambda(b) \) and equal to them, to zero,

\[ \lambda(b) \frac{\partial \lambda(b)}{\partial \lambda(b)} = -\lambda(b) \sum_{d=1}^{D} p(b,d) + \lambda(b) \sum_{d=1}^{D} \sum_{b=1}^{B} \lambda(b)p(b,d) = 0 \] (5)

This leads to the MLEM algorithm as proposed by Shepp and Vardi [4]:

\[ \lambda^{(k+1)}(b) = \frac{\lambda^{(k)}(b)}{\sum_{d=1}^{D} p(b,d) \sum_{b=1}^{B} \lambda^{(k)}(b)p(b,d)} \sum_{d=1}^{D} n^*(d)p(b,d) \] (6)

This reduces the reconstruction time for all subsequent iterations. The reduction is a function of the total number of pixels that are zeroed.

At the early time of reconstruction, the image is not stabilized, applying the threshold might cause the nullification of meaningful pixels. If threshold is applied too late, it saves not much the reconstruction time. In this study, the thresholding technique is applied after 5 iterations. After the thresholding, most non-active pixels are set to zero and there needs no thresholding step again in the succeeding iterations.

C. Thresholding Level

In the following, we will show that thresholding technique can be used to remove noisy parts of an image without affecting image quality. Figure 1 shows the thresholding results for a Derenzo phantom from a PET study. Pixels with their values less than threshold are nullified. The thresholds are set to be \( c^* m \), where \( m \) is the mean pixel value of the whole image and \( c = 1, 2, \text{ and } 4 \). At low threshold, the low-activity parts were removed are mostly outside the phantom. At a higher level of threshold, it can be seen that parts of the removed areas are inside the phantom. These areas are located at cold spots whose activities are known to be zero. Figure 2(a) shows a typical SPECT image from a clinical case. Figure 2(b) shows the images with thresholding levels \( c = 0.8 \). Figure 2(c) is the thresholded image. It can be seen that most of the noisy backgrounds outside the head are removed. Those parts inside the head are at the eyes' locations where the activities should be negligible.
The thresholding level used is dependent upon the image content. For the phantom study, the contrast in the phantom is very high and the thresholding level can be set at higher level. While for the human study the activity distribution of the pixels is less contrasted, i.e. there are many regions with small activities. The thresholding level employed should be small to avoid throwing away useful information.

D. Image Quality Evaluation

The noise, contrast recovery coefficient (CRC) [13], and mean squared errors (MSE) are used to evaluate the quality of reconstructed images. The noise is measured as the standard deviation of the pixel values inside a 5×5 window in the center of the phantom (background). The CRC is defined by:

\[ CRC = \frac{I - \overline{I}}{\overline{I} - B} \]  

where \( I \) and \( \overline{I} \) (\( \overline{B} \) and \( \overline{B} \)) are the average activity at the hot spots (background) in the reconstructed image and digital phantom, respectively. The MSE is defined as the mean of the squared activity differences between the reconstructed image and the digital phantom. It measures the accuracy of the reconstruction technique.

E. Image Data

Two simulated PET study are used to demonstrate the validity of the technique. The simulation is performed using the SIMSET (Simulation System for Emission Tomography) Monte Carlo code from the University of Washington. A phantom of cylindrical shape (radius = 5 cm) containing 3 cylindrical sources (radius = 1.25 cm) is used for simulation. The concentrations of activity (in arbitrary unit) in the three sources are 1000, 1500, and 2000, respectively, and the background activity is 500. All cylinders and the phantom are 20 cm long. A data set with average count of about 770,000 from five measurements are used for the study.

To test the influence of thresholding to the cold spot, a second phantom is employed. The phantom is of elliptical column shape (long axis = 13.13 cm, short axis = 9.38 cm) containing three cylindrical sources with the activity concentration equal to 30, 60, and 100%, respectively. The radii of the three sources are 1.6, 1.6, and 3.4 cm, respectively. The background activity in the phantom is 15% of that in the hot cylinders. A cold spot (radius = 1.6 cm) without any activity is located inside the hottest cylinder. Average count from three measurements equal to 740,000 is generated for this phantom.

Simulation data of both phantoms are acquired for 180 views with each projection consists of 180 bins. Both images are reconstructed with 128×128 pixels.

Results

Figure 3 shows the comparison of reconstructed images obtained after 20 iterations for the data set with \( c = 0.9 \) and without thresholding. Figure 4 plots the horizontal profiles through the images in Figure 3 at the position of \( y = 54 \). The pixels outside the phantom are all removed in the thresholding technique.
both MLEM (with and without thresholding) converge towards a minimum [14,15] after certain iteration number depending on the noise levels in the projection data and then the reconstructions get more noisy afterward. The thresholding technique reduces MSE.

The MSE and noise of the reconstructed image with various thresholding levels are plotted in Figure 6. The reconstruction performance improves with the increase of the thresholding level and the MSE reaches minimum for $c$ value equaling to about 1. However, when the thresholding level is set too high, some of the structural pixels will be eliminated too and the quality of reconstructed image starts to deteriorate. Noise increases as the thresholding is applied. This is due to the increasing value of pixels inside the object. However, the amount of increase in noise is less dependent upon the thresholding level.

The contrast recovery of the reconstructed image with various thresholding levels is plotted in Figure 7. The contrast improves slowly with the increase of the thresholding levels. This is due to the increase of the pixel values inside the phantom and subsequently, the numerator in Eq (8).

Figure 8 plots the MSE of the reconstructed image as a function of time when the thresholding is applied. There is not much difference in the reconstructed images at different application time. In other words, timing for the thresholding
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(a) (b)

Figure 9. Comparison of the reconstructed images obtained after 20 iterations for the second phantom with and without thresholding. Thresholding was applied once after the $5^{th}$ iteration.

<table>
<thead>
<tr>
<th>Iteration No</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80 (w/o thresholding)</td>
</tr>
<tr>
<td>2</td>
<td>79 (thresholding)</td>
</tr>
<tr>
<td>3</td>
<td>78 (thresholding)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>41</td>
<td>55 (thresholding)</td>
</tr>
</tbody>
</table>

Figure 10. The MSE measured with and without thresholding for the second phantom. Thresholding was applied after the $5^{th}$ iteration with level $c=0.9$.

Discussion

The pixel values in PET or SPECT images represent the activity distribution which reflects the metabolism function of the body. Since the functionality of each tissue varies a lot, the activity distribution is highly contrasted. In other words, the difference between object and its background are large and the thresholding technique can easily remove non-active pixels without degrading quality. This is very different for images of structural modalities where the pixel values, representing the physical presentation of that tissue, change smoothly. Of course, further investigations are needed before its clinical application.

MLEM have several advantages over analytical reconstruction algorithms, because many imaging physics, such as non-uniform attenuation and scatter, can be modeled. The incorporation of thresholding technique into MLEM is very simple and most importantly, it does not violate the physical models it describes.

It has been shown that thresholding technique is an efficient way to improve reconstruction performance. The main issue in this study is how large a threshold can be employed without affecting clinical information. Since the total area of active regions is small and the contrast in the activity distribution is quite high, it should be quite safe to employ a large threshold. In this study, we find that even a conservative small thresholding value ($c=0.9$) yields a good result. In the worse case, one can always start over the MLEM using a smaller threshold.

This method is similar to the approach of using active areas and volumes by Li et al [16]. In the active areas approach, computation time is reduced by reconstructing only those pixels which $a$ priori have been determined to possibly contain non-negligible source concentrations of activity. They use a threshold (0.03 times the maximum of projection data) to define the active region. A rectangular reconstruction region is determined that contains all the projection data with intensity greater than the threshold. Their method applies only to rectangular areas outside the body. The proposed method determines the negligible sources based on the pixel values regardless of their locations and thus is more flexible and efficient.

Conclusions

In this paper, we present a method incorporating the thresholding technique into MLEM algorithm to remove non-active background. This simple technique helps accelerate the convergence, reduce computation time, and improve image contrast and reconstruction accuracy. This technique can be easily applied to other iterative reconstruction algorithms like MAP or OSEM.

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