Use of Nakagami Distribution and Logarithmic Compression in Ultrasonic Tissue Characterization

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Received 30 Mar 2006; Accepted 16 May 2006

Abstract

Our previous simulation study showed that the Nakagami parameter estimated using ultrasonic backscattered envelopes compressed by logarithmic computation, denoted by \( m_{log} \), is more sensitive than the original Nakagami parameter \( m \) calculated using uncompressed envelopes for detecting the variation of scatterer concentration in tissues. This study made measurements on phantoms in order to further verify the performance of \( m_{log} \) in quantifying the properties of biological tissues. The ultrasonic backscattered signals from phantoms with different scatterer concentrations were acquired using 5 MHz focused and non-focused transducers. The Hilbert transform and logarithmic compression were in turn applied to the backscattered signals to obtain the uncompressed and compressed envelopes for estimating \( m \) and \( m_{log} \). The experimental results showed that, for both focused and non-focused transducers, the \( m_{log} \) parameter is indeed more sensitive than the \( m \) parameter in differentiating various scatterer concentrations. This may assist in the classification of scatterer properties using the Nakagami statistical model.

Keywords: Nakagami model, Ultrasonic backscattering, Logarithmic compression

Introduction

The Nakagami statistical model, which was initially proposed to describe the statistics of returned radar echoes, has subsequently been extensively applied for tissue characterization by ultrasound [1]. Using this distribution, it is both more general and simpler to model the probability density function (PDF) of the envelope of ultrasonic signals from tissues than other statistical models. The PDF of the Nakagami distribution \( f(r) \) calculated from the backscattered envelopes \( R \) is given by

\[
f(r) = \frac{2m^m r^{2m-1} e^{-\frac{mr^2}{\Omega}}}{\Gamma(m)\Omega^m} U(r),
\]

where \( \Gamma(\cdot) \) and \( U(\cdot) \) are the gamma and unit-step functions, respectively. Two of the parameters, the Nakagami parameter \( m \) and the scaling parameter \( \Omega \), can be calculated using

\[
m = \frac{[E(R^2)]^2}{E[R^2 - E(R^2)]^2}
\]

and

\[
\Omega = E(R^2),
\]

where \( E(\cdot) \) is the statistical mean. The \( m \) parameter is particularly useful for characterizing the probability distributions of ultrasonic backscattered envelopes, including the statistical conditions for pre-Raleigh, Rayleigh, and post-Rayleigh distributions. When the resolution cell of the ultrasonic transducer contains a large number of randomly distributed scatterers, the envelope statistics of the ultrasonic backscattered signals obeys the Rayleigh distribution. If the resolution cell contains the scatterers that have randomly varying scattering cross sections with a comparatively high degree of variance, the envelope statistics are pre-Rayleigh distributions. As the resolution cell contains the periodically located scatterers in addition to the randomly distributed scatterers, the envelope statistics are post-Rayleigh distributions. Because the values of \( m \) ranging from 0 to 1 reflect statistics ranging from pre-Rayleigh to Rayleigh distributions, and that those higher than 1 correspond to post-Rayleigh PDFs, thus the Nakagami parameter can be used to classify the properties of tissues. This has been validated in computer simulations [1,2], experiments on phantoms [1,2], and clinical measurements [3].

In order to better characterize tissues using the Nakagami parameter, some factors that may affect its estimation, such as the pulse length, beam width, attenuation, and background noise, have been explored and discussed [2,4]. Moreover, the effect of the nonlinear logarithmic compression, which is routinely used in existing ultrasonic scanners to adjust the dynamic range of envelope image, on the estimation of Nakagami parameter has also been investigated using computer simulations [5]. The simulation results showed that

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the logarithmic compression would shift the statistics of the backscattered envelopes toward post-Rayleigh distributions for most scatterer concentrations. Furthermore, the Nakagami parameter calculated using compressed backscattered envelopes, denoted by $m_{\log}$, is more sensitive than that calculated using uncompressed envelopes to variations in the scatterer concentration. To further verify the influence of logarithmic compression on the Nakagami parameter in detecting variations in scatterer concentration, the ultrasonic backscattering from phantoms with different scatterer concentrations was measured in this study.

### Materials and methods

Figure 1 shows the experimental setup, including the ultrasonic phantoms, 5 MHz nonfocused (Panametrics-NDT, A309R) and focused (Panametrics-NDT, V309) transducers, a pulser/receiver (Panametrics-NDT, 5072PR), a 500 MHz A/D converter (Signatec, PDA-500), and a personal computer (with 667-MHz Pentium III processor). The experimental procedure is described as follows. Ultrasonic phantoms with various scatterer concentrations were made by adding different weights of glass beads (Supelco, 59200U) with an average diameter of 75 µm into a rectangular plexiglass container filled with distilled water. Scatterer solution (200 ml) added to the container was stirred using a magnetic stirrer to prevent sedimentation and ensure a random distribution of glass beads. The solution concentration ($C$) in the container, which is expressed here as the number of scatterers per cubic millimeter, can be straightforwardly estimated to be

$$C = \frac{m \cdot r^3}{\pi/4 \cdot \gamma \cdot \rho \cdot V}$$

(4)

where $m$, $r$, and $\rho$ correspond to the mass, radius, and density of glass beads, respectively, and $V$ denotes the total volume of the solution in the container.

Before the experiments, the acoustic characteristics of the focused and non-focused transducers were measured using a hydrophone mounted on a positioner controlled by a three-dimensional stepping motor. The aperture size, focal length, and other characteristics of the transducers are listed in Table 1. Both the transducer and the scatterer-medium container were located in a distilled water bath maintained at room temperature. The pulser/receiver was used to excite the transducer transmitting acoustic pulses and to receive backscattered signals that were amplified using a built-in 59-dB gain variable amplifier. An electronic limiter was applied in front of the on-board A/D converter for protection purposes. In the experiments, the ultrasonic backscattered signals were collected from phantoms with scatterer concentrations ranging from 2 to 32 scatterers/mm$^3$. Five samples were made for each scatterer concentration. For each measurement, 256 A-lines of backscattered signals were acquired at the focal zone of the transducer and digitized at a sampling rate of 200 MHz, in which each A-line corresponds to a gated window of 5 µsec. Subsequently, the signal decay of each backscattered echoes due to the diffraction effect of transducer was corrected using the measured axial beam profile. The Hilbert transform was used to obtain the envelopes of the diffraction-compensated backscattered signals, and the envelope-peak method was applied for compensating the effect of acoustic attenuation on the envelope signals [6]. The compressed backscattered envelopes $Z$ for different scatterer concentrations were calculated using

$$Z = \log_{10}(R + 1),$$

(5)

Consequently, the Nakagami parameters before and after logarithmic compression as a function of scatterer concentration were obtained from the estimations of the uncompressed and compressed backscattered envelopes in equation (2).

### Results

Figure 2 shows the experimentally estimated values of $m$ and $m_{\log}$ for the non-focused transducer as functions of scatterer concentrations ranging from 2 to 32 scatterers/mm$^3$. Figures 3 and 4 respectively reveal that the PDFs of backscattered envelopes corresponding to 2 and 32 scatterers/mm$^3$ before and after logarithmic compression when the non-focused transducer is used. For the same increase in the scatterer concentration, the mean value of $m$ varied from 1.17 to 1.31 and the mean value of $m_{\log}$ increased from 5.03 to 8.73, indicating that the statistics of uncompressed backscattered envelopes are close to Rayleigh distribution

| Table 1. Characteristics of the transducers used in the experiments. |
| --- | --- | --- |
| Frequency | 5 MHz | 5 MHz |
| Diameter | 12.7 mm | 12.7 mm |
| Pulse length | 0.85 mm | 0.35 mm |
| $-3$ dB bandwidth | 4.17–5.44 MHz | 3–6.98 MHz |
| Focal length | 13.8 cm | 2 cm |
| $-3$ dB beamwidth | 3.6 mm | 0.8 mm |
and that the statistical distributions of compressed envelopes are different types of post-Rayleigh distribution. Figure 5 shows both $m$ and $m_{\log}$ to be functions of scatterer concentration when the focused transducer was used to measure the ultrasonic backscattered signals from phantoms. Figures 6 and 7 show the statistics of uncompressed and compressed envelopes for scatterer concentrations of 2 and 32 scatterers/mm$^3$ obtained using the focused transducer. The mean value of $m$ increased from 0.27 to 0.72 when the scatterer concentration increased from 2 to 32 scatterers/mm$^3$, indicating that the statistical distribution of the uncompressed envelopes of ultrasonic signals varied from pre-Rayleigh to nearly Rayleigh distribution. On the other hand, when the scatterer concentration increased from 2 to 32 scatterers/mm$^3$ the mean value of $m_{\log}$ increased from 1.66 to 6.8, corresponding to different degrees of post-Rayleigh distributions.

**Discussion and conclusions**

Ultrasound will be scattered if its wavelength is comparable to or greater than the dimensions of scatterers in biological tissues [7]. The resultant backscattered echoes are dependent on the shape, size, concentration, density, and other properties of scatterers [8-13]. Thus, the Nakagami parameter estimated from the second and fourth statistical moments of the backscattered envelope may provide additional information associated with the local microstructures of tissues that will assist in clinical diagnoses.
Figure 5. Nakagami parameter estimated using the focused transducer as a function of scatterer concentration before and after logarithmic compression.

Figure 6. PDFs of backscattered envelope measured using a focused transducer before logarithmic compression. (a) 2 scatterers/mm$^3$; (b) 32 scatterers/mm$^3$.

Figure 7. PDFs of backscattered envelope measured using a focused transducer after logarithmic compression. (a) 2 scatterers/mm$^3$; (b) 32 scatterers/mm$^3$.

Many studies have explored methods for improving the performance of the Nakagami parameter in characterizing biological tissues. To date, these methods can be divided into two main types. The first is the use of a well-focused transducer in the measurement of ultrasonic backscattering [2].

Due to the statistics of the envelope of ultrasonic echoes being determined by the number of scatterers in the resolution cell, the volume of the resolution cell strongly influences estimations of the ultrasonic statistical parameter. As mentioned in the introduction above, when the resolution cell contains a small number of scatterers, the statistics of backscattered envelopes would follow pre-Rayleigh PDFs, corresponding to Nakagami parameters lower than 1. The PDFs for the envelopes of the ultrasonic signals would follow Rayleigh statistics when the number of scatterers in the resolution cell increases to 10 or more [14-16], conforming to the unity Nakagami parameter. Consequently, using a non-focused transducer with a larger resolution cell to measure the backscattered signals, the backscattered statistics from low to high scatterer concentrations would follow a Rayleigh distribution. This is because the large number of scatterers in a larger resolution cell satisfies the condition of the Rayleigh statistics. On the other hand, when a focused transducer is used, the number of scatterers in the resolution cell would gradually increase with the increase in the scatterer concentration of the tissue, making the backscattered statistics vary from pre-Rayleigh to Rayleigh distribution. On this condition, the Nakagami parameter becomes more sensitive to variations in
the scatterer concentration. This has now been verified both by our previous study [2] and from the results of the $m$ parameter as a function of scatter concentration in Fig. 2 and 5.

The second method for improving the sensitivity of Nakagami parameter is to develop new algorithms. Dumane and Shankar developed a modified Nakagami parameter, denoted as $m_{\text{eff}}$, based on the technique of frequency diversity and compounding to enhance the sensitivity of the Nakagami parameter in distinguishing different scatterer concentrations [17]. The frequency diversity was used to create multiple envelopes that were subsequently weighted and combined to form a compound envelope for the estimation of $m_{\text{eff}}$ using equation (2). Although the $m_{\text{eff}}$ parameter has better sensitivity to quantify scatterer concentrations, using $m_{\text{eff}}$ tends to have two disadvantages: (i) the comparatively large amount of computation required for frequency diversity and compounding, and (ii) the sensitivity of the parameter needing to be optimized depending on the associated algorithmic settings (e.g., the bandwidth of the applied filter). These complications limit further applications of that technique.

The experimental results obtained in this study represent that the logarithmic transform is a simple and practical approach with a less computation for enhancing the sensitivity of Nakagami parameter in differentiating variations in scatterer concentrations, as demonstrated by the results of the $m_{\text{log}}$ parameter in Figs. 2 and 5. For a nonfocused transducer, the dynamic range of the $m$ parameter between the lowest and highest scatterer concentration, defined as $\Delta m$, was 0.14, and such a low sensitivity makes it difficult to differentiate different scatterer concentrations. In contrast, the value of $\Delta m_{\text{log}}$ was about 3.7. This much larger value indicates that $m_{\text{log}}$ is much better for detecting variations in the scatter concentration than is $m$. Moreover, for the focused transducer, the dynamic ranges of $m$ and $m_{\text{log}}$ to the scatterer concentrations were 0.45 and 5.14, respectively. It also indicates that the sensitivity of the Nakagami parameter could be enhanced after using the logarithmic transform on the backscattered envelope. The reason why the $m_{\text{log}}$ parameter is more sensitive to the variation of scatterer concentration is that the logarithmic compression would change the backscattered statistics for each scatterer concentration into different degrees of post-Rayleigh distribution, as indicated by Figs. 4 and 7, leading to a higher average slope for the curve of Nakagami parameter as a function of scatterer concentration. However, it should be noted that the dynamic ranges of the $m_{\text{log}}$ parameter obtained from the experiments on phantoms clearly differ from that of our previous simulation study, in which the simulated $m_{\text{log}}$ parameter increased from 0.72 to 2.5 with increasing the scatterer concentration from 2 to 32 scatterers/mm$^2$ (the dynamic range of the $m_{\text{log}}$ parameter is 1.8) [5]. Although the two trends obtained from the computer simulations and the phantom experiments are similar, the inconsistency between their results implies that there should be other factors that could affect the estimation of the $m_{\text{log}}$ parameter. For this reason, we will further investigate the standard procedure for estimating a robust $m_{\text{log}}$ parameter, and this may let $m_{\text{log}}$ be a better Nakagami-based parameter for ultrasonic tissue characterization.

References